

ACH2053 – Introdução à Estatística

Exercícios recomendados

Capítulo 1: Introdução à Probabilidade

Fonte(Morris DeGroot, Mark Schervish. Probability and Statistics. 4th Ed.)

Seção 1.4:

Atenção: No livro do DeGroot, a notação “ \subset ” equivale a “ \subseteq ”

1. Suppose that $A \subset B$. Show that $B^c \subset A^c$.
6. Suppose that one card is to be selected from a deck of 20 cards that contains 10 red cards numbered from 1 to 10 and 10 blue cards numbered from 1 to 10. Let A be the event that a card with an even number is selected, let B be the event that a blue card is selected, and let C be the event that a card with a number less than 5 is selected. Describe the sample space S and describe each of the following events both in words and as subsets of S :

- a. $A \cap B \cap C$
- b. $B \cap C^c$
- c. $A \cup B \cup C$
- d. $A \cap (B \cup C)$
- e. $A^c \cap B^c \cap C^c$.

7. Suppose that a number x is to be selected from the real line S , and let A , B , and C be the events represented by the following subsets of S , where the notation $\{x: - - -\}$ denotes the set containing every point x for which the property presented following the colon is satisfied:

$$A = \{x: 1 \leq x \leq 5\},$$

$$B = \{x: 3 < x \leq 7\},$$

$$C = \{x: x \leq 0\}.$$

Describe each of the following events as a set of real numbers:

- a. A^c
- b. $A \cup B$
- c. $B \cap C^c$
- d. $A^c \cap B^c \cap C^c$
- e. $(A \cup B) \cap C$.

11. A power cell consists of two subcells, each of which can provide from 0 to 5 volts, regardless of what the other subcell provides. The power cell is functional if and only if the sum of the two voltages of the subcells is at least 6 volts. An experiment consists of measuring and recording the voltages of the two subcells. Let A be the event that the power cell is functional, let B be the event that two subcells have the same voltage, let C be the event that the first subcell has a strictly higher voltage than the second subcell, and let D be the event that the power cell is not functional but needs less than one additional volt to become functional.

- a. Define a sample space S for the experiment as a set of ordered pairs that makes it possible for you to express the four sets above as events.
- b. Express each of the events A , B , C , and D as sets of ordered pairs that are subsets of S .
- c. Express the following set in terms of A , B , C , and/or D : $\{(x, y) : x = y \text{ and } x + y \leq 5\}$.
- d. Express the following event in terms of A , B , C , and/or D : the event that the power cell is not functional and the second subcell has a strictly higher voltage than the first subcell.

Seção 1.5:

1. One ball is to be selected from a box containing red, white, blue, yellow, and green balls. If the probability that the selected ball will be red is $1/5$ and the probability that it will be white is $2/5$, what is the probability that it will be blue, yellow, or green?

2. A student selected from a class will be either a boy or a girl. If the probability that a boy will be selected is 0.3, what is the probability that a girl will be selected?

3. Consider two events A and B such that $\Pr(A) = 1/3$ and $\Pr(B) = 1/2$. Determine the value of $\Pr(B \cap A^c)$ for each of the following conditions: **(a)** A and B are disjoint; **(b)** $A \subset B$; **(c)** $\Pr(A \cap B) = 1/8$.

4. If the probability that student A will fail a certain statistics examination is 0.5, the probability that student B will fail the examination is 0.2, and the probability that both student A and student B will fail the examination is 0.1, what is the probability that at least one of these two students will fail the examination?
5. For the conditions of Exercise 4, what is the probability that neither student A nor student B will fail the examination?
6. For the conditions of Exercise 4, what is the probability that exactly one of the two students will fail the examination?
8. If 50 percent of the families in a certain city subscribe to the morning newspaper, 65 percent of the families subscribe to the afternoon newspaper, and 85 percent of the families subscribe to at least one of the two newspapers, what percentage of the families subscribe to both newspapers?
10. For two arbitrary events A and B , prove that

$$\Pr(A) = \Pr(A \cap B) + \Pr(A \cap B^c).$$

Seção 1.6:

1. If two balanced dice are rolled, what is the probability that the sum of the two numbers that appear will be odd?
2. If two balanced dice are rolled, what is the probability that the sum of the two numbers that appear will be even?
3. If two balanced dice are rolled, what is the probability that the difference between the two numbers that appear will be less than 3?
4. A school contains students in grades 1, 2, 3, 4, 5, and 6. Grades 2, 3, 4, 5, and 6 all contain the same number of students, but there are twice this number in grade 1. If a student is selected at random from a list of all the students in the school, what is the probability that she will be in grade 3?
5. For the conditions of Exercise 4, what is the probability that the selected student will be in an odd-numbered grade?

6. If three fair coins are tossed, what is the probability that all three faces will be the same?
8. Consider an experiment in which a fair coin is tossed once and a balanced die is rolled once.
 - a. Describe the sample space for this experiment.
 - b. What is the probability that a head will be obtained on the coin and an odd number will be obtained on the die?

Seção 1.7:

1. Each year starts on one of the seven days (Sunday through Saturday). Each year is either a leap year (i.e., it includes February 29) or not. How many different calendars are possible for a year?
2. Three different classes contain 20, 18, and 25 students, respectively, and no student is a member of more than one class. If a team is to be composed of one student from each of these three classes, in how many different ways can the members of the team be chosen?
3. In how many different ways can the five letters a , b , c , d , and e be arranged?
4. If a man has six different sportshirts and four different pairs of slacks, how many different combinations can he wear?
5. If four dice are rolled, what is the probability that each of the four numbers that appear will be different?
6. If six dice are rolled, what is the probability that each of the six different numbers will appear exactly once?
7. If 12 balls are thrown at random into 20 boxes, what is the probability that no box will receive more than one ball?
8. An elevator in a building starts with five passengers and stops at seven floors. If every passenger is equally likely to get off at each floor and all the passengers leave independently of each other, what is the probability that no two passengers will get off at the same floor?

9. Suppose that three runners from team A and three runners from team B participate in a race. If all six runners have equal ability and there are no ties, what is the probability that the three runners from team A will finish first, second, and third, and the three runners from team B will finish fourth, fifth, and sixth?

10. A box contains 100 balls, of which r are red. Suppose that the balls are drawn from the box one at a time, at random, without replacement. Determine (a) the probability that the first ball drawn will be red; (b) the probability that the 50th ball drawn will be red; and (c) the probability that the last ball drawn will be red.

Seção 1.8:

1. Two pollsters will canvas a neighborhood with 20 houses. Each pollster will visit 10 of the houses. How many different assignments of pollsters to houses are possible?

2. Which of the following two numbers is larger: $\binom{93}{30}$ or $\binom{93}{31}$?

3. Which of the following two numbers is larger: $\binom{93}{30}$ or $\binom{93}{63}$?

4. A box contains 24 light bulbs, of which four are defective. If a person selects four bulbs from the box at random, without replacement, what is the probability that all four bulbs will be defective?

6. Suppose that n people are seated in a random manner in a row of n theater seats. What is the probability that two particular people A and B will be seated next to each other?

7. If k people are seated in a random manner in a row containing n seats ($n > k$), what is the probability that the people will occupy k adjacent seats in the row?

8. If k people are seated in a random manner in a circle containing n chairs ($n > k$), what is the probability that the people will occupy k adjacent chairs in the circle?

9. If n people are seated in a random manner in a row containing $2n$ seats, what is the probability that no two people will occupy adjacent seats?

- 10.** A box contains 24 light bulbs, of which two are defective. If a person selects 10 bulbs at random, without replacement, what is the probability that both defective bulbs will be selected?
- 11.** Suppose that a committee of 12 people is selected in a random manner from a group of 100 people. Determine the probability that two particular people A and B will both be selected.
- 12.** Suppose that 35 people are divided in a random manner into two teams in such a way that one team contains 10 people and the other team contains 25 people. What is the probability that two particular people A and B will be on the same team?
- 13.** A box contains 24 light bulbs of which four are defective. If one person selects 10 bulbs from the box in a random manner, and a second person then takes the remaining 14 bulbs, what is the probability that all four defective bulbs will be obtained by the same person?
- 16.** The United States Senate contains two senators from each of the 50 states. **(a)** If a committee of eight senators is selected at random, what is the probability that it will contain at least one of the two senators from a certain specified state? **(b)** What is the probability that a group of 50 senators selected at random will contain one senator from each state?
- 17.** A deck of 52 cards contains four aces. If the cards are shuffled and distributed in a random manner to four players so that each player receives 13 cards, what is the probability that all four aces will be received by the same player?
- 18.** Suppose that 100 mathematics students are divided into five classes, each containing 20 students, and that awards are to be given to 10 of these students. If each student is equally likely to receive an award, what is the probability that exactly two students in each class will receive awards?

19. A restaurant has n items on its menu. During a particular day, k customers will arrive and each one will choose one item. The manager wants to count how many different collections of customer choices are possible without regard to the order in which the choices are made. (For example, if $k = 3$ and a_1, \dots, a_n are the menu items, then $a_1a_3a_1$ is not distinguished from $a_1a_1a_3$.) Prove that the number of different collections of customer choices is $\binom{n+k-1}{k}$. *Hint:* Assume that the menu items are a_1, \dots, a_n . Show that each collection of customer choices, arranged with the a_1 's first, the a_2 's second, etc., can be identified with a sequence of k zeros and $n - 1$ ones, where each 0 stands for a customer choice and each 1 indicates a point in the sequence where the menu item number increases by 1. For example, if $k = 3$ and $n = 5$, then $a_1a_1a_3$ becomes 0011011.

Seção 1.9:

- 1.** Three pollsters will canvas a neighborhood with 21 houses. Each pollster will visit seven of the houses. How many different assignments of pollsters to houses are possible?
- 2.** Suppose that 18 red beads, 12 yellow beads, eight blue beads, and 12 black beads are to be strung in a row. How many different arrangements of the colors can be formed?
- 3.** Suppose that two committees are to be formed in an organization that has 300 members. If one committee is to have five members and the other committee is to have eight members, in how many different ways can these committees be selected?
- 4.** If the letters $s, s, s, t, t, t, i, i, a, c$ are arranged in a random order, what is the probability that they will spell the word "statistics"?
- 5.** Suppose that n balanced dice are rolled. Determine the probability that the number j will appear exactly n_j times ($j = 1, \dots, 6$), where $n_1 + n_2 + \dots + n_6 = n$.
- 6.** If seven balanced dice are rolled, what is the probability that each of the six different numbers will appear at least once?

7. Suppose that a deck of 25 cards contains 12 red cards. Suppose also that the 25 cards are distributed in a random manner to three players *A*, *B*, and *C* in such a way that player *A* receives 10 cards, player *B* receives eight cards, and player *C* receives seven cards. Determine the probability that player *A* will receive six red cards, player *B* will receive two red cards, and player *C* will receive four red cards.

8. A deck of 52 cards contains 12 picture cards. If the 52 cards are distributed in a random manner among four players in such a way that each player receives 13 cards, what is the probability that each player will receive three picture cards?

9. Suppose that a deck of 52 cards contains 13 red cards, 13 yellow cards, 13 blue cards, and 13 green cards. If the 52 cards are distributed in a random manner among four players in such a way that each player receives 13 cards, what is the probability that each player will receive 13 cards of the same color?

10. Suppose that two boys named Davis, three boys named Jones, and four boys named Smith are seated at random in a row containing nine seats. What is the probability that the Davis boys will occupy the first two seats in the row, the Jones boys will occupy the next three seats, and the Smith boys will occupy the last four seats?